Numerical demonstration of spatial photonic Ising machine with parallel processing based on spatial multiplexing

Suguru Shimomura, Yusuke Ogura, and Jun Tanida

Graduate School of Information Science and Technology, Osaka University, 1-5 Yamadaoka, Suita, Osaka, 565-0871, Japan

Author e-mail address: s-shimomura@ist.osaka-u.ac.jp

Abstract: In this study, we propose a method for implementing the spatial photonic Ising machine with parallel processing capability. By using spatial multiplexing, the Ising Hamiltonian at different spin states can be computed simultaneously. The feedback based on the solutions obtained from the parallel processing provides efficient searches for the optimal solution. To evaluate the performance of the proposed system, we searched for the optimal solution of Max-cut problems. We confirmed that the solution distribution is improved by the spatial multiplexing.

1. Introduction

Solving combinational optimization problems is useful in a wide range of fields including finance, pharmacy, and engineering. An Ising machine is a computational system designed to search for the optimal solution [1]. Individual problems are formulated as Ising problems, and the choices and requirements in the individual problems are represented as spin states taking the value of ±1 and their interactions, respectively. The optimal solution is derived by exploring the minimum value of the Ising Hamiltonian. The computation of the Ising Hamiltonian can be implemented by utilizing physical phenomena, and various studies for implementing the Ising machines as hardware are proposed. A spatial photonic Ising machine (SPIM), in which the spins are encoded into the phase distribution of light, has the ability to handle more than 4×10^4 spins and to search for the solution in a large scale of combinational optimization problems [2]. Moreover, the calculation time to obtain the Ising Hamiltonian remains even if the number of spins increases. In the SPIM, iterative calculation of the Ising Hamiltonian is required to evaluate the obtained solution. The number of solutions increases exponentially with the number of spins, and therefore the number of iterations to find the optimal solution increases. On the other hand, the iteration speed is limited by the operating speed of electronic devices including a spatial light modulator (SLM). To accelerate the time for solving the problem by the SPIM, optical methods for parallel processing are a promising way to calculate the Ising Hamiltonians simultaneously.

In this study, we propose the SPIM with parallel processing function by spatial multiplexing. Multiple spin states are encoded into the phase distribution. Demultiplexing with designed phase allows acquisitions of the Ising Hamiltonians simultaneously. Moreover, updating the spin states based on the obtained Ising Hamiltonians contributes to search of the optimal solution efficiently.

2. Parallel processing by using spatial multiplexing

Figure 1 shows a scheme of the parallel processing method. The interaction between the spins is encoded into an amplitude distribution, which consists of the duplication of the pattern expressing $\boldsymbol{\xi}$. The phase distributions of the light are modulated by a SLM to binary phase $\phi(\boldsymbol{s}) \in \{-\pi, 0\}$ to represent spin states as $\boldsymbol{\sigma} = \exp\{i\phi(\boldsymbol{s})\} \in \{-1, 1\}$. For

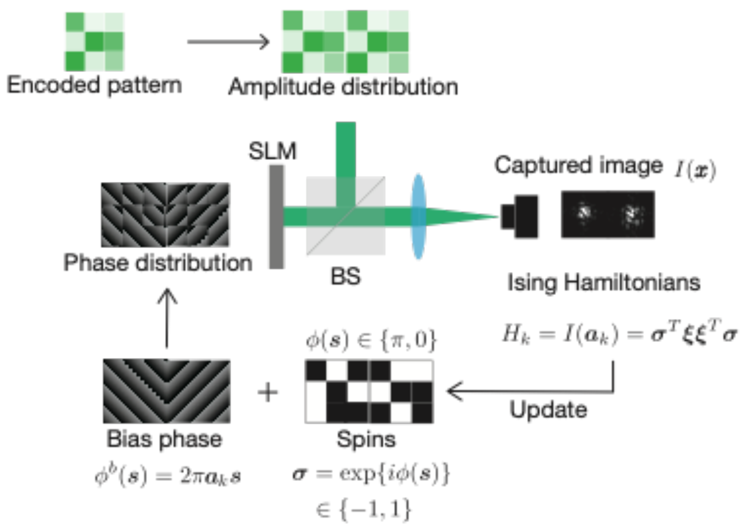

Fig 1 Schematic of parallel processing with SPIM using spatial multiplexing.

parallel processing, the multiple type of spin states $\boldsymbol{\sigma}_k$ are prepared as phase patterns. The individual phase patterns are added with bias phase $\boldsymbol{\phi}_k^b(\boldsymbol{s}) = 2\pi a_k \boldsymbol{s}$, which consists of grating patterns. Here, a_k is an angle coefficient of the *k*-th grating. After modulation by the SLM, light patterns are focused by a lens, and the intensity distribution $I(x)$ is captured by an image sensor. The captured image can be divided into multiple light patterns depending on the directions of each grating. The intensity value at each area is equal to the Ising Hamiltonian H_k and represented as

$$I(a_k) = H_k = \boldsymbol{\sigma}_k^T \boldsymbol{\xi}\boldsymbol{\xi}^T \boldsymbol{\sigma}_k.$$

As a result, several types of Ising Hamiltonians H_k at the spin state σ_k are obtained simultaneously. Owing to the parallel processing, the number of Ising Hamiltonians obtained by the SPIM at constant time increases without the iteration. Moreover, the candidates of spin states can be determined from multiple results, and the efficient search for the optimal solution can be achieved.

3. Searching performance in Max-cut problem using numerical simulation

To evaluate the performance of the proposed method, the Max-cut problem was solved using the numerical simulation based on optical propagation. The Max-cut problem is a well-known benchmark task of the Ising machine, and the optimal solution is searched by formulating the Ising Hamiltonian as $H = -\boldsymbol{\sigma}^T \boldsymbol{\xi}\boldsymbol{\xi}^T \boldsymbol{\sigma}$. In the numerical simulation, the number of multiplexing was set to three, and the simulated annealing was used as the algorithm for finding the optimal solution. Figure 2 (a) shows evolutions of the Ising Hamiltonian depending on the number of iterations. The number of the spins and the iterations were 400 and 900, respectively. As the iteration and the intensity values I_k increased, the Ising Hamiltonian decreased and converged to the minimum value. From the minimum Ising Hamiltonian, we confirmed that the optimized solution was obtained. Moreover, we investigated the dependence between the search performance and the number of multiplexing. Lower Ising Hamiltonians were obtained by increasing the number of multiplexing. Figure 2 (b) shows histograms of the accuracy at obtained solution. The numerical simulation with individual multiplexing was run 50 times. In the case of without multiplexing, the median value of the accuracy was 0.88, and the optimized solution could not be obtained. The accuracy distributions were improved by increasing the number of multiplexing, and the median value of the accuracy was 0.99 when the number of multiplexing was 3. These results show that the use of multiplexing improves the performance to find the optimized solutions.

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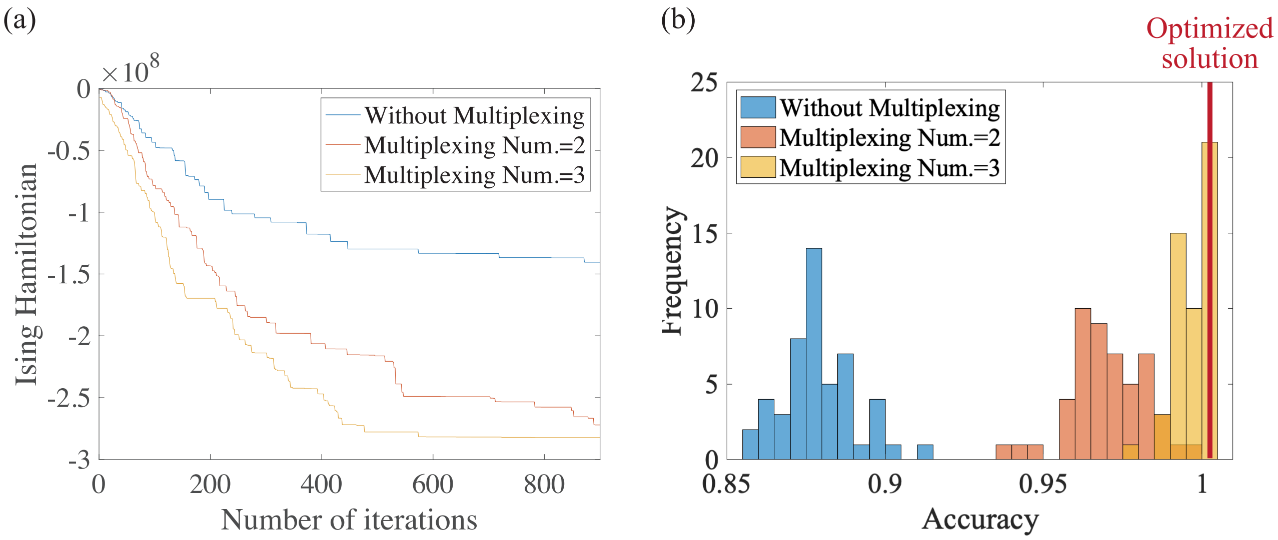

Fig 2 (a) Evolution of Ising Hamiltonian at each number of multiplexing. (b) Histogram of the accuracy of the obtained solution depending on the number of multiplexing.

References

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