Design of quantum dot networks for improving the performance of reservoir computing

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Abstract: Quantum dot networks (QDNs), which generate a variety type of the fluorescence signal depending on their structure, are useful for physical reservoir computing. However, random signals generated by QDNs have been used in quantum dot reservoir computing (QDRC). In this study, we proposed a method for designing QDNs' structures which generate effective fluorescence signals to improve the performance of the QDRC. We evaluated the signal diversity of the designed QDNs and the prediction performance for Santa Fe time-series data.

1. Introduction

A quantum dot (QD), which is a nano meter-sized fluorophore, transfers its own energy to neighbor ones by Förster resonance energy transfer (FRET) within several distances. The fluorescence signal generated from QDs can be modulated depending on FRET efficiency. Quantum dot networks (QDNs), which are constructed by multiple-step FRETs in randomly-distributed QDs, have the ability to generate a variety of fluorescence signal depending on the network structure. Generation of diverse signals in time domain is useful for implementation of physical reservoir computing (RC) which predicts time-series data [1]. We aim to the construction of a quantum dot reservoir computing (QDRC) system and demonstated that QDN has an echo-state property which is a necessary function in RC [2]. However, the relationship between the signals generated by the QDNs and the prediction performance in the QDRC remains unknown. To maximize the performance of QDRC, it is necessary to construct QDNs generating effective fluorescence signals for the prediction. In this study, we propose a method for design of QDNs' structure to generate a variety of temporal signals for RC. By evaluating the independence of the signals generated by the numerous QDNs' structures, an effective set of signals for the prediction is chosen. To verify the effectiveness of designed QD structures, we generate the temporal signals by numerical simulation and evaluates the performance in prediction of chaotic signals.

2. The mathematical model of the QDN and the method of designing QDNs

First, the mathematical model of the QDN is described. Considering the electrons in a QD as a two-level system, the rate equation for the excited electrons in the i-th QD is written as follows:

$$\frac{dN_{e,i}}{dt} = \frac{\sigma_i I_{ex}(t)}{h\nu_i} N_{g,i} - k_{nr,i}N_{e,i} - k_{r,i}N_{e,i} - \sum_{i\neq j} k_{i\to j}N_{e,i} + \frac{N_{g,i}}{N_{e,i}+N_{g,i}} \sum_{i\neq j} k_{j\to i}N_{e,j}\,. \tag{1}$$

Here, $N_{e,i}, N_{g,i}$ are the number of electrons at the excited and ground states in the QDs. $I_{ex}(t), h$ are the irradiation photon density at time t, the Planck constant, respectively. The absorption coefficient and the frequency of fluorescence of i-th QD is represented as σ_i, ν_i. k_r and k_{nr} are rate constants of radiative and nonradiative relaxation processes, and $k_{i\to j}$ is rate constants of FRET from i-th to j-th QD. The fluorescence signal at time t generated from the QDN is

$$f(t) = \sum_i \left(k_{r,i}N_{e,i}(t) \times h\nu_i\right) \;\; (t = 1, \dots, T). \tag{2}$$

Equations (1) and (2) can be used to calculate the time-series signal $\boldsymbol{f} = \left(f(1), \dots, f(T)\right)^{\top}$ generated by the QDN. Therefore, the fluorescence signals in the QDNs can be simulated by them.

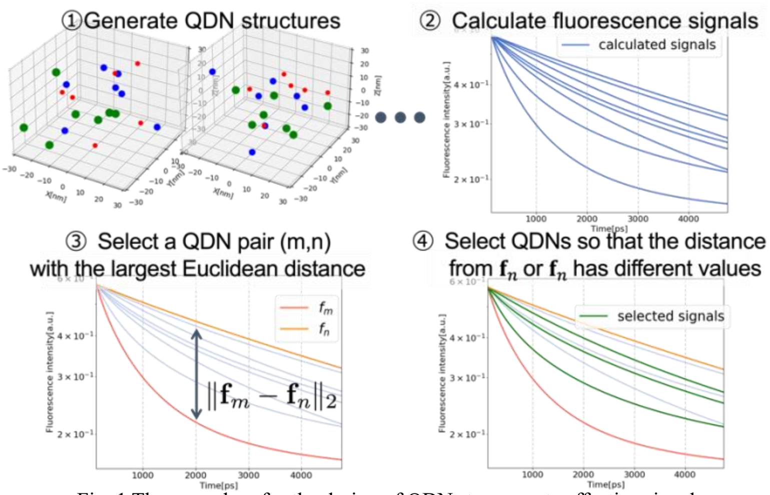

Fig. 1 The procedure for the design of QDNs to generate effective signals.

Figure 1 shows the procedure for the design of QDNs that provide effective signals. Initially, we randomly generated N types of QDN structures. Individual fluorescence signals $\boldsymbol{f}_i$ $(i = 1, \cdots, N)$ of the i-th QDN are calculated by Eqs. (1) and (2). The fluorescence signals are integrated as a matrix $\boldsymbol{F} = (\boldsymbol{f}_1, \cdots, \boldsymbol{f}_N)$. The rank value of the matrix $\boldsymbol{F}$ corresponds to the dimension of the output value. In RC, the predicted signal is regressed by the linear combination of the output value. Therefore, the effective output values for RC are determined by the combination of QDNs with a high rank of the matrix $\boldsymbol{F}$. To select such a combination, we chose the pair of fluorescence signals $\boldsymbol{f}_m$ and $\boldsymbol{f}_n$that maximizes the Euclidean distance $\|\boldsymbol{f}_i - \boldsymbol{f}_j\|_2$ between the two signals $\boldsymbol{f}_i$ and $\boldsymbol{f}_j$. In addition, we selected the remaining QDNs so that the distance from $\boldsymbol{f}_m$ or $\boldsymbol{f}_n$ took different values. The various type of signals can be chosen by the proposed method.

3. Evaluation by the rank of the fluorescence matrix

To investigate the effectiveness of the proposed method, we evaluated them by the rank of fluorescence matrix. In this study, we generated 3000 types of QDN structures, and selected 18 types of QDNs. Figure 2(a) shows the temporal signals generated from all types of QDNs and the selected 18 types of QDNs. A variety of fluorescence signals can be selected. The rank of the matrix $\boldsymbol{F}_m$, consisting of the selected signals, was 14. Figure 2(b) shows the frequency distribution of the ranks of the matrix $\boldsymbol{F}_{\mathrm{random}}$, when 18 types of the fluorescence signals were selected randomly. The average rank for a random selection was 5.14, and the maximum value of the rank was 10 in 1000000 trials. This result shows that the QDNs with effective fluorescence signal for the construction of the RC could be designed.

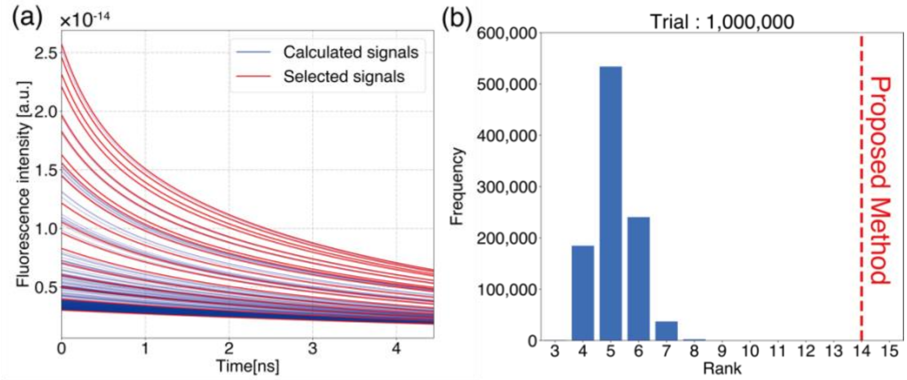

Fig. 2. (a) The temporal signals from the selected 18 types of QDNs (red) and all types of QDNs (blue). (b) The frequency distribution of the ranks of the matrix $\boldsymbol{F}_{\mathrm{random}}$.

4. Evaluation by the performance of quantum dot reservoir computing

Next, we investigated the well-designed effective QDNs improves the prediction performance of the QDRC. A one-step-ahead prediction was performed on Santa Fe time-series data by using designed QDNs. The number of training and test data was 800, and the Santa Fe signal was normalized by the maximum value. The prediction performance was evaluated using the normalized mean square error (NMSE).

Figures 3(a) and (b) show the prediction results of the Santa Fe signal with QDRC composed of a randomly selected QDNs (R-QDRC) and the designed QDNs (D-QDRC), respectively. The prediction result of R-QDRC is shown in Fig. 3(a). The NMSE calculated from prediction results of R-QDRC is 2.53×10^{-1}. In contrast, the NMSE of D-QDRC shown in Fig. 3(b) was 1.91×10^{-2}, which is smaller than R-QDRC. This result shows that use of the designed QDN improve the performance of RC.

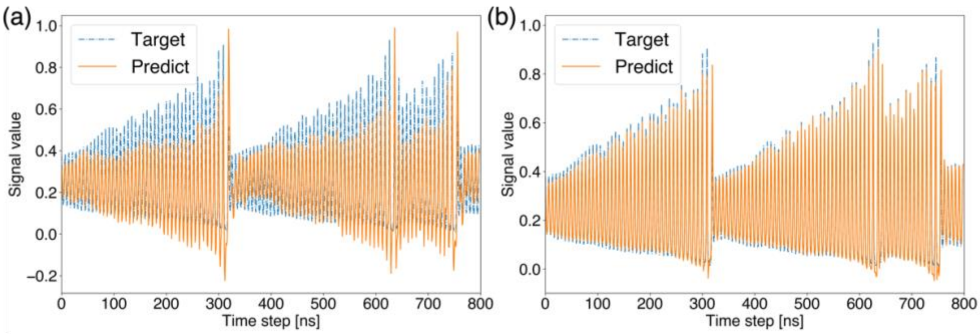

Fig. 3. Prediction results of Santa Fe signal. (a) R-QDRC. (b) D-QDRC.

Acknowledgements

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