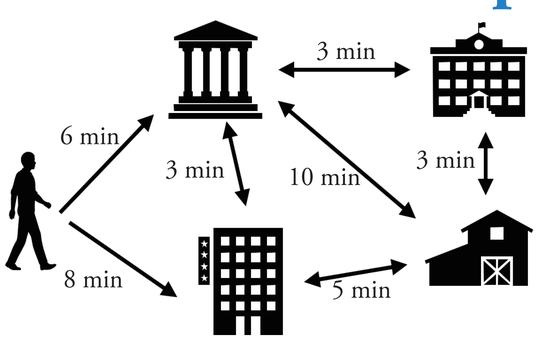
Parallel processing of spatial photonic Ising machine by spatial multiplexing

Suguru Shimomura

Graduate school of Information Science and Technology, Osaka University

E-mail: s-shimomura@ist.osaka-u.ac.jp

Combinational optimization problems



What is best route in shortest time?

Finding a best combination from many choices.

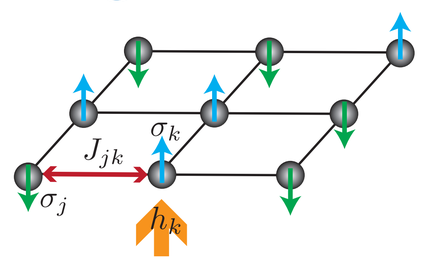
Ex. Traveling salesman problem, knapsack problem

They are known as NP-hard problem.

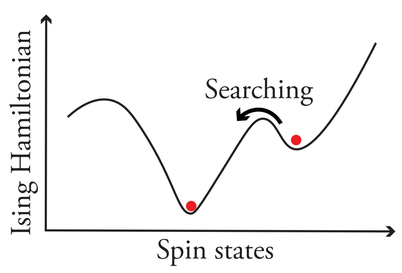
Number of spots	Number of combinations
4	3
8	2520
16	6.5×10^{11}
32	4.1×10^{33}
64	9.9×10^{86}

Number of combinations increases exponentially.

Ising model



Interaction: $\boldsymbol{J}$
Spin states: $\boldsymbol{\sigma}$
External magnetic field: $\boldsymbol{h}$



Problems are represented as the model of magnetic material.

Solutions are evaluated as Ising Hamiltonian

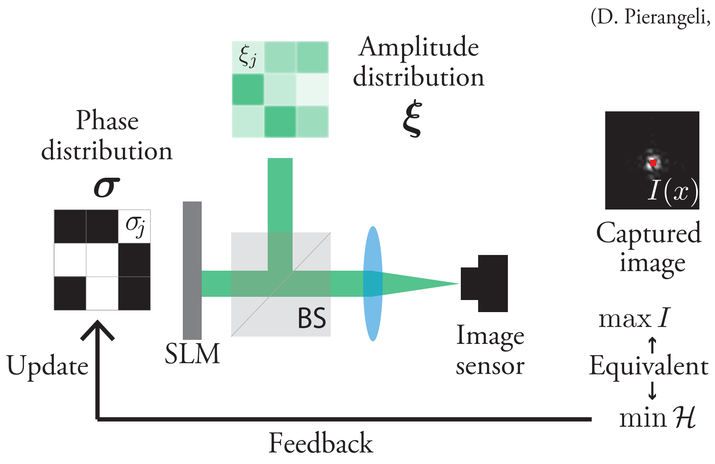
$$\mathcal{H} = -\sum_{j,k} \boldsymbol{J}\sigma_j\sigma_k - \sum_j h_j\sigma_j$$

Combination and conditions is encoded into spin states and interactions.

Optimal solutions are obtained by minimizing the Ising Hamiltonian.

Spatial photonic Ising machine (SPIM)

(D. Pierangeli, *et al*, Phys. Rev. Lett., 122(21), 213902, 2019)



Spins are encoded into a phase distribution

Ising Hamiltonian:

$$\mathcal{H} = -I(0) = -\sum_{j,\ k} \xi_j\xi_k\sigma_j\sigma_k$$

$$J_{j,k} \propto \xi_j\xi_k$$

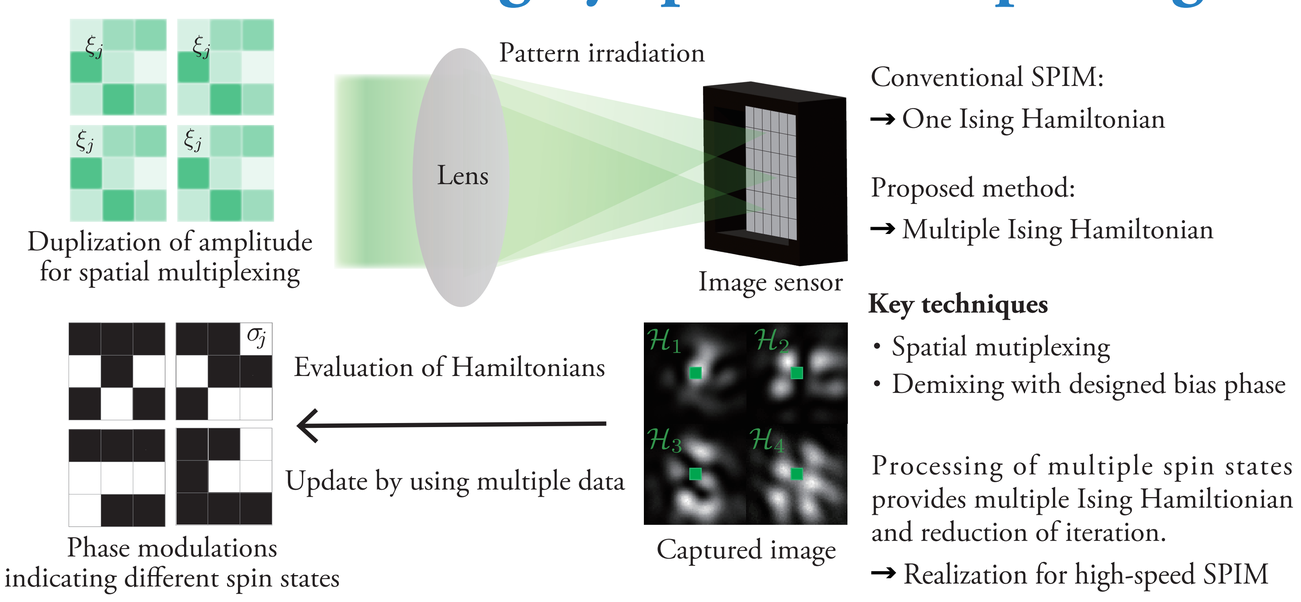
Phase update so that intensity is maximized.

Features

- Number of Spin: 8.0×10^4
- Constant calculation time

Feedback and update speed : 60~120 Hz
Enormous iterations for searching the solutions ➔ Limitation of processing time in SPIM.

Parallel Processing by spatial multiplexing



Conventional SPIM:
➔ One Ising Hamiltonian

Proposed method:
➔ Multiple Ising Hamiltonian

Key techniques

- Spatial mutiplexing
- Demixing with designed bias phase

Processing of multiple spin states provides multiple Ising Hamiltionian and reduction of iteration.
➔ Realization for high-speed SPIM

Demixing with bias phase

0. Output pattern of SPIM

$$I(\boldsymbol{x}) = \sum_{j,\ h} \xi_j\xi_h\sigma_j\sigma_h\delta_W^2(\boldsymbol{x})\exp\{2iW(h-j)\boldsymbol{x}\}$$

$\delta_W(\boldsymbol{x})$: Sinc function $\boldsymbol{x}$: coordinates of output pattern

W : Pixel pitch of SLM

1. Bias phase with grating pattern

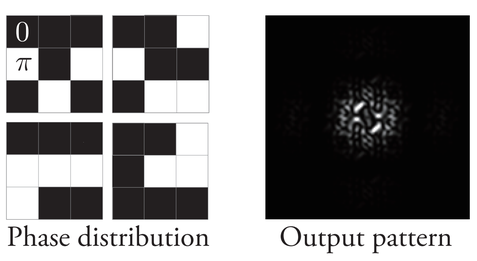
$\phi_{\mathrm{bias},j} = 2\pi\alpha j$ α : Angle of grating

2. Redefinition of spins

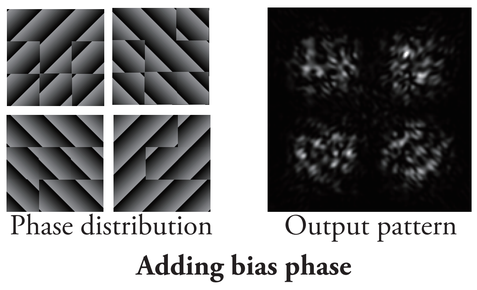
$\sigma_j = \exp\{i(\phi + \phi_{\mathrm{bias,j}})\}$ $\phi \in \{0, \pi\}$

3. Shifting position of the Ising Hamiltonian

$$I(\alpha) = \mathcal{H} = \sum_{j,\ h} \xi_j\xi_h\sigma_j\sigma_h$$

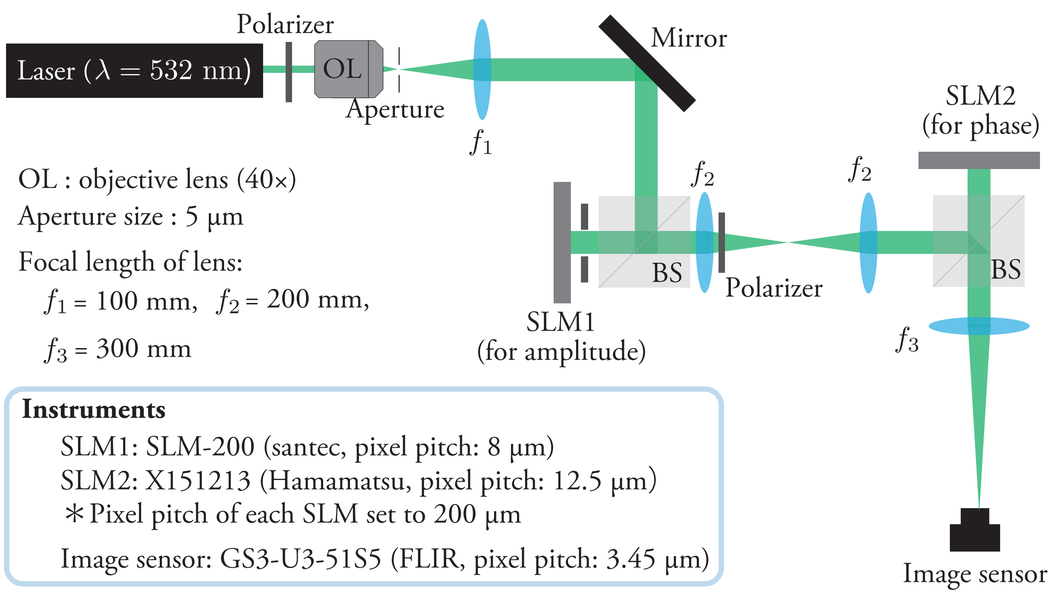


Without adding bias phase



Adding bias phase

Optical setup



OL : objective lens (40×)
Aperture size : 5 μm
Focal length of lens:
f_1 = 100 mm, f_2 = 200 mm,
f_3 = 300 mm

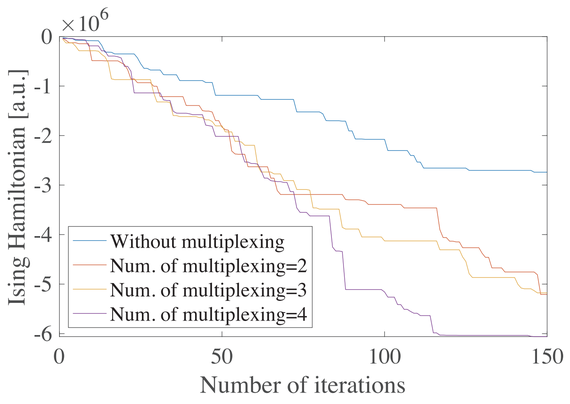
Instruments

SLM1: SLM-200 (santec, pixel pitch: 8 μm)
SLM2: X151213 (Hamamatsu, pixel pitch: 12.5 μm)
＊Pixel pitch of each SLM set to 200 μm
Image sensor: GS3-U3-51S5 (FLIR, pixel pitch: 3.45 μm)

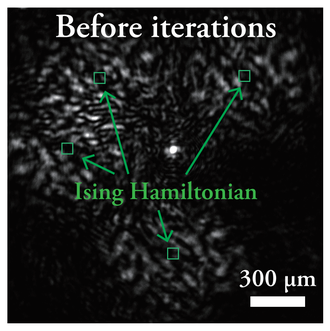
Searching result by using SPIM with spatial multiplexing

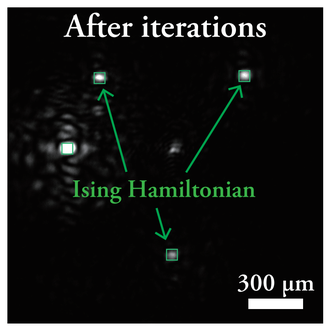
Parameters

Number of spins: 100
Number of iterations : 150
Number of trials: 50
Optimization algorithm: Simulated annealing
Amplitude: $\xi_j \in \{0, 1\}$



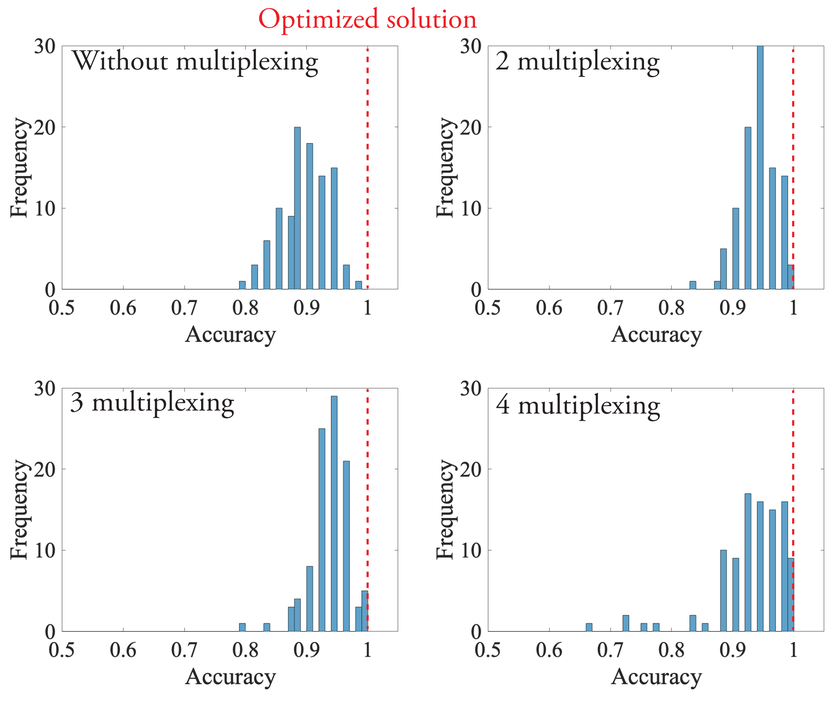
Evolution of the Ising Hamiltonian depending on he number of multiplexing





Captured images

Optimized solution



Histograms of the obtained solutions

Number of multiplexing	Probability for obtaining the optimized solution
Without	0.0 %
2	3.0 %
3	5.0 %
4	9.0 %

Results

- Intensity inceased by iterations.
- Ising Hamiltonians decreased rapidly.
- Solution distribution become better.
- Accuracies increased by spatial multiplexing.

We demonsrated that the use of spatial multiplexing enhance an capability to search the optimal solution.

Summary

- The spatial photonic Ising machine enables to solve the optimal solutions of the combinational optimization problems.
- By using spatial multiplexing, multiple Ising Hamiltonians at individual spin states are obtained simltaneously.
- Ising Hamiltonian decreased more by using spatial multipexing
- The number of trials in which the optimal solutions are obtained increased depending on the number of multiplexing.

Acknoledgements

This resarch is supported by

- JSPS KAKENHI (Grant Number 23H04805)
- JST-ALCA-Next Program (Grant Number JPMJAN23F2)
- CREST (Grant Number JPMJCR18K2)