Spatial photonic Ising machine by binary phase encoding with DMD

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Abstract: ***We present a spatial photonic Ising machine with binary phase control using a DMD, aiming at high-speed optical computing for combinatorial optimization problems. The experimental result demonstrates the effectiveness of the method.***
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I. INTRODUCTION

Combinatorial optimization problems are widely relevant to our daily lives, such as transportation and investment portfolios. However, when large-scale problems are handled, combinatorial explosion occurs, and it takes an enormous amount of time to obtain acceptable solutions. To address this issue, Ising machines, which are dedicated computing systems to solve combinatorial optimization problems, have been actively studied. The Ising machine searches for an optimal solution based on the Ising model, a mathematical model expressing the behavior of magnetic materials' spins. Ising machines have been implemented by using, for example, trapped ions [1], superconducting circuits [2], and CMOS circuits [3].

Ising machines based on photonics are also promising to solve large-scale optimization problems, because the use of light provides the capability in high-speed parallel processing. Photonic Ising machines include a coherent Ising machine [4], where degenerate optical parametric oscillators are used. A system that handles 100,000 spins with all-to-all interactions has been realized using an FPGA [4]. Another notable photonic Ising machine is a spatial photonic Ising machine (SPIM), which searches for the optimal solution based on optical modulation [5].

By using spatial information, the computing system becomes highly scalable in the number of spins, and the spins are implemented without wiring for the interaction in the Ising model. These strengths indicate the high potential of the SPIM. The SPIMs generally use a liquid crystal type of spatial light modulators (SLMs) to modulate amplitude and phase for encoding the problem and combination, respectively; however, the modulation rate of a liquid-crystal SLM is typically an order of ten Hz, and this has been a limiting factor in increasing the computing speed. In this study, we propose an SPIM using a digital micromirror device (DMD), the switching speed of which is more than tens of thousands Hz. In addition to the phase control, we realize the representation of a given optimization problem by controlling each mirror of the DMD in a suitable manner. This strategy enables implementation of the SPIM using a single DMD alone as a modulator. The validity of the proposed method is demonstrated by performing a solution search for a number partitioning problem as an optimization problem example.

II. BINARY PHASE CONTROL METHOD USING DMD

A DMD consists of an array of micromirrors which can be switched between two tilt-angle states (ON and OFF), and it is usually used as a binary-amplitude modulator. In this study, binary phase control is implemented by a specialized way for usage of the DMD. For achieving phase control, a set of micromirrors are required to be used as a single pixel. To deal with larger scale optimization problems, a minimized number of micromirrors should be assigned to individual pixels. We thus adopted a method where two mirrors are used as a pair and only one of them is switched to the ON state to express the binary phase. Figure 1 shows the scheme of the phase control method. By setting the incident angle to the micromirrors appropriately, the optical path difference between the light reflected by the adjacent mirrors is adjusted to half a wavelength. This configuration makes it possible to express the phase $\{0, \pi\}$ by switching the ON-state mirror.

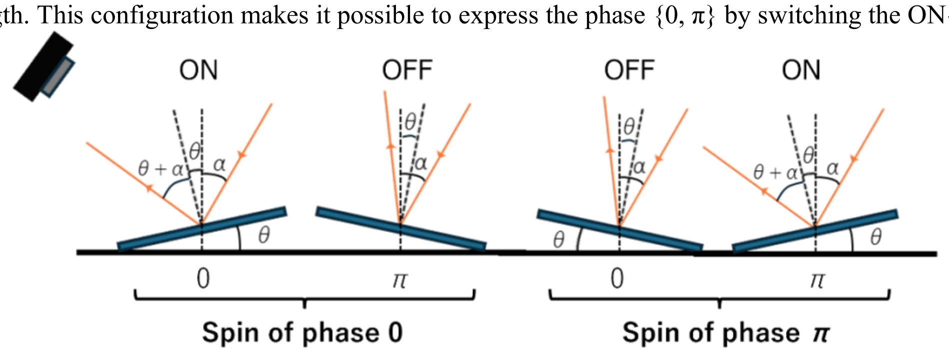

Fig. 1. Scheme of the phase control method with a DMD

III. Experimental setup and solution search procedure

The optical setup of the SPIM with a DMD is shown in Fig. 2. The light emitted from the He-Ne laser (wavelength: 632.8 nm) is collimated through a spatial filter and Lens 1 and reflected by Mirror 1. After passing through the aperture, the light reflected by Mirror 2 hits the DMD (ViALUX V9001).
The light reflected by the DMD is Fourier transformed by Lens 4, and the light intensity distribution at the focal plane of Lens 4 is captured using an image sensor (Eosens1.1MCX12-CM). The angle of light incident to the DMD is set to 10.302° for binary phase control. A high-speed image sensor with a maximum acquisition rate of 3,674 Hz was used, so that the image sensor was not the limiting factor for the computing speed of the system. The captured image is processed using a computer, and the modulation pattern of the DMD is updated based on the result.

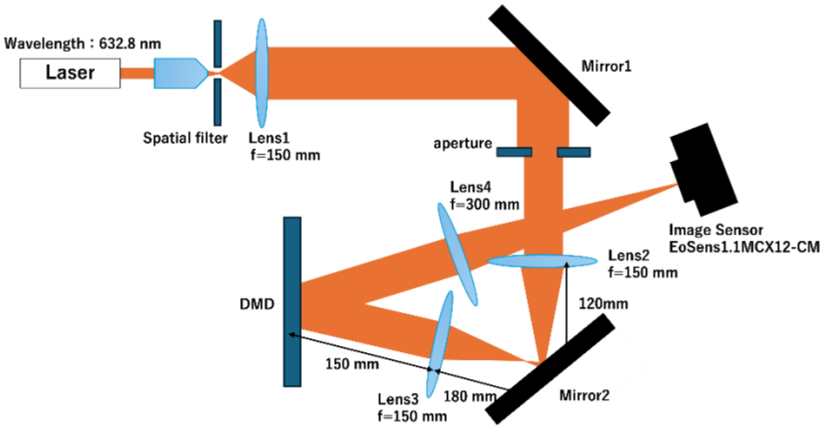

Fig. 2. Optical setup.

The operation of the SPIM with a DMD is formulated as follows. Spatially uniform light with a unit amplitude hits the DMD. For simplicity, let us consider only one-dimensional coordinate. The spin state $\sigma_j \in \{-1, 1\}$ is linked to the binary phase $\phi_j \in \{0, \pi\}$ controlled by the mirrors, and it is represented as $\sigma_j = e^{i\phi_j}$. In this system, amplitude modulation is not applied, and the light wavefield $E(k)$ reflected from the DMD is expressed as

$$E(k) = \sum_j \sigma_j \tilde{\delta}(k - k_j), \qquad (1)$$

where $\tilde{\delta}(x)$ is a rectangular function. The light wave expressed in Equation (1) propagates through the lens, and the light intensity distribution $I(x)$ is acquired by the image sensor on the focal plane. Because $I(x)$ is calculated as the square of the absolute value of the inverse Fourier transform of Equation (1),

$$I(x) = \sum_{j,h} \sigma_j \sigma_h \delta_W^2(x) e^{2iW(h-j)x}, \qquad (2)$$

Where W is half the pitch size of the micromirror and $\delta_W(x)$ is the inverse Fourier transform of $\tilde{\delta}_W(k)$. Considering the center position, $x = 0$, of the light intensity distribution on the image sensor, $e^{2iW(h-j)x} \sim 1$ and $\delta_W(x) \sim 1$, and then Equation (2) becomes

$$I(0) = \sum_{j,h} \sigma_j \sigma_h \,. \qquad (3)$$

The given combinatorial optimization problem is encoded by the amplitude modulation. Let $N_{j'}$ be the amplitude corresponding to spin j'. In our method, $N_{j'}$ pixels are used as a group and their phases are controlled with synchronization to represent the amplitude $N_{j'}$. Equation (3) can then be rewritten into Equation (4):

$$I(0) = \sum_{j',h'} N_{j'} N_{h'} \sigma_{j'} \sigma_{h'}. \qquad (4)$$

This equation corresponds to the Hamiltonian $H = -\sum_{j',h'} J_{j'h'} \sigma_{j'} \sigma_{h'}$ of the Mathis model when $J_{j'h'} = N_{j'} N_{h'}$, indicating that the solution search for the combinatorial optimization problems represented by this model is possible.

IV. Solution Search Results

To demonstrate the effectiveness of our method, we conducted a solution search for a number partitioning problem. In this problem, a given multiset of numbers is partitioned into two subsets such that the summations of the numbers in the individual subsets are the same. The multiset used in the experiment has 14 elements {1, 4, 5, 5, 6, 6, 6, 7, 7, 7, 8, 10, 12, 14, 15, 60}, so that in the optimal solution, two subsets have a total summation of 80. The number of iterations was set to be 500, and simulated annealing was used as the solution search algorithm. Figure 3 shows the change of the sum of the numbers in a subset during the solution search. 20 solution search trials were conducted. It can be seen that in all trials, the obtained solution approaches to one of the optimal solutions in which the summation is 80. The difference between the summations in one subset in the obtained final solution and that in the optimal solution is shown in Fig. 4. In all trials, the difference between the summations is 2 or less. In addition, the optimal solution was obtained in 10 trials. The experimental result demonstrates the effectiveness of the method.

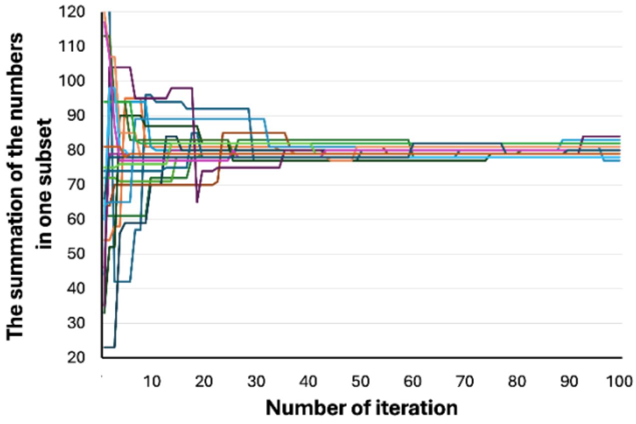

Fig. 3. Change in the summation of the numbers in one subset.

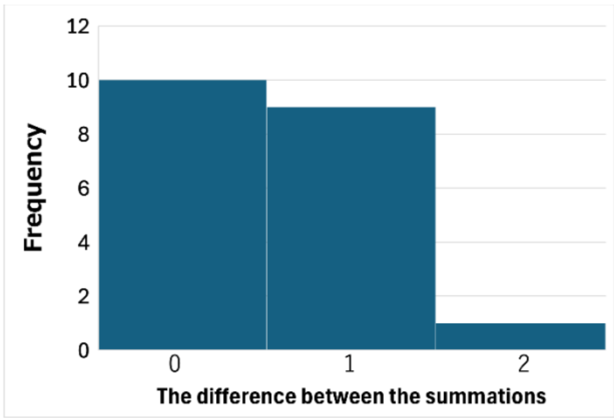

Fig. 4. Histogram of the difference between the summation of one subset in the obtained solution and that in the optimal solution.

V. CONCLUSION

We presented an SPIM using a DMD, which is switchable at a high speed. In our method, amplitude and phase modulation to encode the given problem and the spin states is controlled by using a single DMD alone. This scheme is helpful to implement high speed computing. We demonstrated the validity of our method by implementing an optical system and solving a number partitioning problem.

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